Revisiting the omitted price bias in the estimation of production functions*

Julia González Cerdeira , Universidad de Vigo
Saul Lach, The Hebrew University of Jerusalem
Daniel Miles, Universidad de Vigo

November 3, 2023

Abstract

We revisit the bias in the estimation of production functions with firm-level data due to the lack of physical quantities on output and inputs. We show that constructing firm-specific prices from available data on firm-specific price changes, and using them to deflate revenues and expenditures, introduces a measurement error into the empirical production function. This error reflects the unobserved base year prices used in the construction of the firm-specific prices. The usual practice of ignoring them generates an omitted variable bias (OVB). Monte Carlo simulations suggest that this bias can be significant. Because of the OVB, the estimates are sensitive to the choice of base year. The OVB disappears in our simulations when firm-specific fixed effects are incorporated into the estimation of the production function.

*We thank Jose Carlos Fariñas, Omar Licandro, Ariel Pakes, Andrea Pozzi, Mark Roberts, Fabiano Schivardi, as well as participants at the IIO Conference (Boston, 2022) and EARIE Conference (Vienna, 2022), for helpful comments and suggestions. An earlier version of this paper entitled "Firm-level price information and the production function estimation" is avaliable at RGEA Working Paper 6, 2013. All errors are our responsibility.

1 Introduction

The presence of unobservables in the estimation of production functions raises a set of empirical challenges that have no straightforward solution (Ackerberg et al., 2007;Pakes, 2021; De Loecker and Syverson, 2021). Broadly speaking, these unobservables can be classified into structural variables, such as unobserved productivity, or measurement problems, such as unobserved firm-level quantities.

Most of the literature focused on addressing the problem of unobserved productivity. As first shown by Marschak and Andrews (1944), when managers make input decisions after observing their firm's productivity, optimal behavior implies a correlation between unobserved (to the researcher) productivity and input choices. This makes the input regressors inherently endogenous in the estimation of production functions.[1]

Measurement problems arise because most firm-level datasets report firm revenues and materials expenditures rather than physical quantities. When revenues and materials expenditures need to be deflated, and firm-specific prices are not observed, an additional source of endogeneity may arise. Klette and Griliches (1996) showed that using aggregate price indices to deflate these nominal variables introduces an omitted variable bias.

In some cases, relying on additional structural assumptions may solve this omitted variable problem.[2] In other cases, however, identification of the production function may require observing the firm's output and input price levels, e.g., when price levels are state variables (e.g., Doraszelski and Jaumandreu, 2013, among others).

Datasets reporting price *levels* are rare, but there is an increasing number of datasets reporting firm-specific price *changes*.[3] Based on these data, the unobserved firm price levels can be recovered by

[1]Different approaches have been proposed to address this endogeneity problem: proxy variable methods initiated by Olley and pakes (1996) (see also Levinsohn and Petrin, 2003; Ackerberg et al., 2015), dynamic panel data models (Blundell and Bond, 2000), and methods exploiting the structural restrictions imposed by profit maximization (e.g, Doraszelski and Jaumandreu, 2013; Grieco et al., 2016; Gandhi et al., 2020.

[2]For example, Klette and Griliches (1996) assume specific demand systems, while De Loecker et al. (2016, 2021) model the unobserved input price as a function of observables.

[3]For example, the Swedish Industry Statistics Survey (Carlsson et al., 2021), the German Manufacturing Sector Micro dataset (Mertens, 2020), the manufacturing firm-level dataset provided by Statistics Denmark (Smeets and Warzynski,

a recursion on the price changes plus an unknown initial condition, the unobserved price level in the base year. This unknown component is essentially a measurement error in the true, but unobserved, price level when it is measured by the price recovered from price change data.

This measurement error – the unobserved price level in the base year – is usually ignored when recovering the firm's specific price level from observed price changes. Specifically, it is standard practice to set the base year (log) price level to zero for all firms in the sample and use the resulting firm-level prices to deflate revenues and expenditures (e.g., Eslava et al., 2004 or Koch et al., 2021, among others).

The main goal of this paper is to show that ignoring the unobserved price in the base year, by normalizing it to zero, generates an *omitted variable* which is likely to bias the estimators of the production function parameters. We show that the base year price is a firm-specific fixed effect which is likely to be correlated with input demands and should therefore be taken into account in the estimation procedure.

Using Monte Carlo simulations, we show that the normalization of the unobserved base price years can generate biases of up to 70 percent in the estimated parameters. These biases do indeed disappear when the measurement error is treated as a firm-specific fixed effect in the empirical production function.[4]

Another effect of ignoring the base year prices is that the estimates change when the base year changes. This is worrisome because the choice of base year in constructing the firm-specific prices is arbitrary and at the discretion of the researcher. To be clear, using the same data but choosing a different base year (e.g., the year at the middle of the sample instead of the initial year) can generate different estimated parameters. We illustrate this problem by using a synthetic dataset of the Spanish Encuesta sobre Estrategias Empresariales (ESEE). The average change (in absolute value) across the ten industrial sectors in the Spanish data ranges between 11 and 33 percent. In some sectors, changing the base year changes the estimates by 97 percent. When using firm-specific fixed effects to capture

2013), the Spanish ESEE manufacturing database (Doraszelski and Jaumandreu, 2013), among others.

[4]In our Monte Carlo simulations we use a log-linear model based on a Cobb-Douglas production function which implies that the measurement error is additively separable. If the production function is non-linear incorporating fixed effects into the estimation procedure may be more challenging, but this is beyond the main goal of this paper.

the unobserved base year the estimated parameters are invariant to the choice of base year.

Finally, we point out that, even when the production function is estimated consistently, residual estimates of firm-specific productivity levels are biased by the presence of base year prices. Although firm-specific productivity growth can still be recovered, aggregate productivity is still biased and this bias is likely to be negative. We show, however, that estimating productivity from the second stage of proxy estimation methods delivers the correct firm-specific productivity levels.

The paper is organized as follows. Section 2 presents the problem of the unobserved base year prices and the bias resulting from their normalization. Section 3 presents the results of Monte Carlo simulations that quantify the magnitude of this bias. Section 4 tackles the issue of the change of base year, while Section 5 points out some implications of the lack of information on base year prices on the estimation of productivity. Conclusions close the paper.

2 The bias from normalizing the unobserved price level in the base year

In this section we derive the empirical production function when physical quantities are measured by deflated revenues and materials expenditures. We show that when firm-specific price levels are recovered form data on price changes, the unknown base year prices appear as additional unobservables in the production function equation.

2.1 The empirical production function

The single-product production function for firm j is $Q_{jt} = F(K_{jt}, L_{jt}, M_{jt})e^{\omega_{jt}+\epsilon_{jt}}$, where ω_{jt} is productivity, K_{jt}, L_{jt}, M_{jt} are measures of capital, labor and materials, and ϵ_{jt} is an I.I.D. zero mean shock to production which is unknown to the firm when making its input decisions.[5]

[5] We focus on a single-product production function. This is the usual case because most datasets do not report product-specific information. Equation (1) may be viewed as a single-product production function in a multi-product firm when the decision to add additional products is independent of the unobserved productivity and/or the firm's use of inputs(De Loecker et al., 2016).

In logarithms, denoted by lower-case letters, the production function can be written as

$$q_{jt} = f(k_{jt}, l_{jt}, m_{jt}) + \omega_{jt} + \epsilon_{jt} \tag{1}$$

To estimate equation (1) we need to address two problems arising from the fact that researchers do not observe all the information used by firms when making their input decisions. The first problem is the non-observability of the firm's productivity, ω_{jt}, while the second problem is the (possible) non-observability of physical quantities of output and inputs.

The unobserved productivity generates an endogeneity problem because it is correlated with input choices. The literature paid special attention to this problem and developed several approaches to deal with this issue (e.g., see references in Syverson, 2011; Ackerberg et al., 2015; De Loecker and Syverson, 2021; Pakes, 2021).

On the other hand, the non-observability of physical quantities, and its measurement by deflated revenues and input expenditures, received much less attention in the literature. It was not until Klette and Griliches (1996) showed that the standard practice of using industry-wide price indexes to deflate firm-level revenues generates an omitted variable (price) bias that this issue came to the attention of empirical researchers.

To overcome this price bias researchers make additional structural assumptions, such as specific demand functions or an input price control function (e.g., Klette and Griliches, 1996; DeLoecker et al. 2016, 2020). In other instances, researchers use alternative sources of price information, such as custom data, to construct proxies for the unobserved firm-specific prices (Morlacco, 2020).

Recently, the increasing availability of firm-level datasets reporting information on firms' year-to-year changes in their output and input prices allows researchers to overcome the lack of full information on prices. These reported price changes are used to recover firm-specific price levels which are then used to deflate nominal quantities. In this Section we analyze this case in detail and spell-out the often implicit assumptions necessary for its implementation.

For clarity of exposition we start by addressing the problem associated with the lack of data on physical output q_{jt} – equivalently, unobserved firm-specific output price levels – and later introduce the additional problem of not observing materials quantity m_{jt}.[6]

Most firm-level datasets report revenues $R_{jt} = Q_{jt}P_{jt}$ instead of physical output Q_{jt}. When we use revenue to measure output, the production function equation becomes,

$$r_{jt} = q_{jt} + p_{jt} = f(k_{jt}, l_{jt}, m_{jt}) + \omega_{jt} + \epsilon_{jt} + p_{jt} \tag{2}$$

where r_{jt} are log revenues and p_{jt} is log output price.

In equation (2), p_{jt} is unobserved and its omission generates an output price bias when it is correlated with input choices. The standard approach to address this issue has been to measure output as deflated revenues, i.e., $r_{jt} - p^d_{jt}$, where p^d_{jt} is a price deflator (in logs). When using an industry-wide price deflator, for example, $p^d_{jt} = p^d_t$ for all firms in the industry, the estimated production function becomes

$$r_{jt} - p^d_t = f(k_{jt}, l_{jt}, m_{jt}) + \omega_{jt} + \epsilon_{jt} + \left(p_{jt} - p^d_t\right) \tag{3}$$

As pointed out by Klette and Griliches (1996), this approach introduces an additional unobservable given by the gap between the firm's price and the industry price, $\left(p_{jt} - p^d_t\right)$. This price gap is likely to be correlated with input demands when prices vary across firms within an industry. Estimators that ignore this unobseved heterogeneity suffer, therefore, from an omitted variable (price) bias.

As mentioned above, one solution to the problem of not observing p_{jt} in (2) is to construct firm-specific prices from reported price changes.[7] Letting $\Delta P_{jt} = \frac{P_{jt} - P_{jt-1}}{P_{jt-1}}$, the unobserved firm price P_{jt} is recovered by using a recursive formula on these price changes and a price level in the base year P_{jb}, where year $b < t$ is the base year,

[6]Throughout the paper we assume that the quantity of labor is observed because most datasets report the number of employees or working-hours.

[7]Examples of papers using price changes to construct a firm-specific price index are Eslava et al. (2004); Mairesse and Jaumandreu (2005); Smeets and Warzynski (2013); Dolado et al. (2016); Jaumandreu and Lin (2018); Mertens (2020); Carlsson et al. (2021); Chen and Steinwender (2021), among others.

$$P_{jt} = P_{jt-1}(1 + \Delta P_{jt}) = P_{jb} \prod_{s=b+1}^{t} (1 + \Delta P_{js}) = P_{jb} \prod_{s=b+1}^{t} \frac{P_{js}}{P_{js-1}}$$

and in logs,

$$p_{jt} = \begin{cases} p_{jb} + \sum_{s=b+1}^{t} \Delta p_{js} & \text{if} \quad t > b \\ \\ p_{jb} & \text{if} \quad t = b \end{cases} \tag{4}$$

where $\Delta p_{js} = \ln\left(\frac{P_{js}}{P_{js-1}}\right) = p_{js} - p_{js-1}$.[8]

Using equation (4) we can rewrite revenues as $r_{jt} = q_{jt} + p_{jb} + \sum_{s=b+1}^{t} \Delta p_{js}$, for $t > b$, and since p_{jb} is unobserved because only price changes are observed, the empirical production function becomes

$$r^*_{jt} \equiv r_{jt} - \sum_{s=b+1}^{t} \Delta p_{js} = q_{jt} + p_{jb} = f(k_{jt}, l_{jt}, m_{jt}) + \omega_{jt} + \epsilon_{jt} + p_{jb} \tag{5}$$

for $t > b$, while for year $t = b$ we have $r^*_{jb} = r_{jb} = q_{jb} + p_{jb}$.

Note that in equation (5) output is measured by *partially deflated revenue* r^*_{jt} because the base year price, p_{jb}, is unobserved by the researcher. Put differently, the base year price can be viewed as a measurement error when physical output is measured by partially deflated revenues, $r^*_{jt} = q_{jt} + p_{jb}$.[9]

2.2 Normalization of the base year output price level

Notice the similarity between equations (3) and (5): both equations include an unobserved firm-specific price. In equation (3) this term is the firm's price relative to the industry-wide price deflator, $p_{jt} - p^d_t$, while in equation (5) it is the firm price level in the base year, p_{jb}. In both cases, the parameter estimators will be biased if they ignore the correlation between these unobservables and input demands.

[8] The base year is arbitrarily chosen by the researcher. For clarity of exposition we focus on the natural case where the base year is the first year the firm appears in the sample, $b \leq t$. When this is not the case, the formula in equation (4) changes in an appropriate way.

[9] The "measurement error" term p_{jb} is also referred interchangeably as an "omitted variable".

These unobservables, however, are treated differently in the literature. The usual approach to overcome the bias due to the unobservables in equation (3) has been to introduce structural assumptions into the estimation. In contrast, the unobserved base year price in equation (5) has been usually ignored by its de-facto normalization to zero. Indeed, the usual solution to the presence of the unobserved base year price level has been to set its value to zero, $p_{jb} = 0$ for all j.

This normalization of the base year price seems restrictive for several reasons. First, it implies that firms have the same exact price level in the common base year and different prices in other years. This implied assumption seems overly restrictive; even more so if we take into account that the base year is chosen by the researcher.

Second, it conflates unobserved firm-specific price levels with observed price changes. Let the price levels set by the firm be $(P_{j1}, P_{j2} ..., P_{jT}) = (P_{j1}, (1 + \Delta P_{j2})P_{j1}, ..., \prod_{s=2}^{T}(1 + \Delta P_{js})P_{j1})$. This equality no longer holds when the researcher chooses an ad-hoc value for the unobserved price level in the first year (the base year) of the sample. That is, setting $P_{j1} = 1$ (as implied by the zero normalization of the log price) implies that $(P_{j1}, P_{j2} ..., P_{jT})$ is no longer necessarily equal to $\left(1, (1 + \Delta P_{j2}), ..., \prod_{s=2}^{T}(1 + \Delta P_{js})\right)$. Thus, the normalized prices does not necessarily generate the correct set of firm-level prices.[10]

Finally, for the seemingly trivial normalization to work it requires a strong behavioral assumption: the lack of correlation between output price in the base year, p_{jb}, and input demands in any year $t \geq b$. Otherwise, there is an omitted variable bias.[11]

[10] Unfortunately, this has lead to some confusion on whether having access to data on firm-level price changes is equivalent to observing the firm's levels of prices. Many papers that use price change data do indeed construct firm-specific price deflators and use them to deflate nominal quantities (instead of using industry-wide price deflators), but they do not seem to be aware of the possible omitted variable bias generated by the normalization of the base year prices.

[11] Doraszelski and Jaumandreu (2013, hereafter DJ) also normalize the base year prices to zero but follow an alternative two-step approach to derive price levels from data on price changes only (developed in Jaumandreu and Lin, 2018). First, they compute the price level for each period t using the same recursion formula, $P_{jt} = P_{jt-1}\left(1 + \Delta P_{jt}\right)$, with $\Delta P_{jt} = \frac{P_{jt} - P_{jt-1}}{P_{jt-1}}$ being observed, while normalizing the base year price to 1. That is, this first step price is $P^*_{jt} \equiv \prod_{s=b+1}^{t}\left(1 + \Delta P_{js}\right) = \frac{P_{jt}}{P_{jb}}$ reflecting the accumulated price changes between the base year and period t. In a second step they normalize P^*_{jt} by the average of its values for each firm, $\overline{P}^*_j \equiv \frac{1}{T}\sum_{t=1}^{T} P^*_{jt} = \frac{\overline{P}_j}{P_{jb}}$, where $\overline{P}_j$ is the *unobserved* price average, $\overline{P}_j = \frac{1}{T}\sum_{t=1}^{T} P_{jt}$. The DJ deflator is then $P^{DJ}_{jt} = \frac{P^*_{jt}}{\overline{P}^*_j} = \frac{P_{jt}}{\overline{P}_j}$, so that the (log) true price is measured with an error, $\log\left(P^{DJ}_{jt}\right) = p_{jt} - \log\left(\overline{P}_j\right)$. Deflating log revenues by the log of this normalized price gives $r^{*DJ}_{jt} = q_{jt} + p_{jt} - \log\left(P^{DJ}_{jt}\right) = q_{jt} + \log\left(\overline{P}_j\right)$, which corresponds to the RHS of equation (5) with the unobserved (log)

2.3 Unobserved input base year price level

An analogous problem appears when the firm's input price level is recovered from input price changes. To be specific, let the log of materials expenditure be $e_{jt} = m_{jt} + p_{Mjt}$, where p_{Mjt} is the unobserved log materials price level. Then, partially deflated materials expenditure is $m^*_{jt} = e_{jt} - \sum_{s=b+1}^{t} \Delta p_{Mjs} = m_{jt} + p_{Mjb}$ where p_{Mjb} is the unobserved base year materials price level (the measurement error).

When the physical quantity of materials is measured by partially deflated materials expenditures the empirical production function becomes

$$r^*_{jt} = f(k_{jt}, l_{jt}, m^*_{jt} - p_{Mjb}) + \omega_{jt} + \epsilon_{jt} + p_{jb}. \tag{6}$$

since $m_{jt} = m^*_{jt} - p_{Mjb}$.

Clearly, not observing the input base year price level introduces an additional unobservable, p_{Mjb}, into the empirical production function. The precise way p_{Mjb} enters equation (6) depends on the functional form of $f(\cdot)$. For example, when the production function is Cobb-Douglas, (6) becomes $r^*_{jt} = \alpha_L l_{jt} + \alpha_M m^*_{jt} + \alpha_K k_{jt} + \omega_{jt} + p_{jb} - \alpha_M p_{Mjb} + \epsilon_{jt}$ so that p_{Mjb} also enters additively. The usual solution has been to ignore this term by normalizing its value to zero, i.e., setting $p_{Mjb} = 0$ for all j.

Summary

The empirical production function obtained when revenues and materials expenditures are deflated using prices recovered from data on price changes includes additional unobservables reflecting base year price levels. Standard practice, however, is to ignore these unobservables by setting them to zero. For these arbitrary normalizations to work, strong behavioral assumptions are needed.

It is sensible to posit that a firm's demand for inputs depends, among other things, on the input prices it faces. These prices may vary across firms because of differences in productivity, location and/or firm-specific random shocks (Grieco et al., 2016; Ackerberg et al., 2015). In this case, p_{Mjb} is

average price taking the place of the base year price. Thus, the problems associated with estimating (5) are also present in the DJ approach.

necessarily correlated with partially deflated materials, one of the regressors, because $m^*_{jt} = m_{jt} + p_{Mjb}$. It may also be correlated with labor demand, l_{jt}. The output price in the base year, p_{jb}, may also be correlated with input demands in other years. The precise type of correlation depends on the model of firm behavior.[12] If the variation across firms in base year prices is ignored, either by assuming they are zero (or any other constant) or by actually treating them as an uncorrelated error components, this is likely to introduce a price bias in the spirit of Klette and Griliches (1996).

In the next section we use Monte Carlo simulations to quantify the impact of ignoring the unobserved base year price levels on estimators of the production function parameters.

3 Monte Carlo simulation results

We first describe the data generating process used in the simulations. We then derive the empirical production function to be estimated with the simulated data. The precise specification depends on the type of price data available to researchers. Finally, we present results from two simulation designs: one where productivity is observed and the other one where it is not.

3.1 Data generation

The data generating process (DGP) follows the basic DGP developed by Grieco et al. (2016; hereafter GLZ) but applied to a Cobb-Douglas production function and with some minor modifications.

The Cobb Douglas production function is,

$$Q_{jt} = e^{\omega_{jt}} K_{jt}^{\alpha_K} M_{jt}^{\alpha_M} L_{jt}^{\alpha_L}$$

[12]For example, if input demand depends directly on output price and the latter is serially correlated, p_{jb} will be correlated with m^*_{jt}. Another example is when output price is itself endogenous and depends on productivity. Because productivity is usually viewed as serially correlated this creates a link between p_{jb}, ω_{jb} and ω_{jt} and, if input demands depend on productivity as well, p_{jb} will likely be correlated with input demand in other years. In Appendix A we use a simple model to formalize some of these intuitive arguments.

The exogenous productivity process is a first-order autoregressive process

$$\omega_{jt} = \rho_0 + \rho_1 \omega_{jt-1} + \epsilon^{\omega}_{jt}.$$

The firm's capital stock is $K_{jt+1} = K_{jt} + I_{jt}$ and evolves over time based on the investment rule $log(I_{it}) = \xi\omega_{jt} + (1-\xi)log(K_{jt})$ which is compatible with the Olley and pakes (1996) assumptions (see Grieco et al., 2016).

The idiosyncratic labor and material (log) input prices are generated by

$$p_{L_{jt}} = \lambda_{L_0} + \lambda_{L_1} p_{L_{jt-1}} + \epsilon^{L}_{jt+1}$$
$$p_{M_{jt}} = \lambda_{M_0} + \lambda_{M_1} p_{M_{jt-1}} + \epsilon^{M}_{jt+1}$$

where $\left(\epsilon^L_{jt}, \epsilon^M_{jt}, \epsilon^{\omega}_{jt}\right)' \sim N(0, \Sigma)$, where Σ is a 3×3 positive matrix.

Departing from the specification used by GLZ we explicitly allow for serial correlation in input prices and for correlations between the shocks to input prices and to productivity, following standard practice in the literature (e.g., Ackerberg et al., 2015; Doraszelski and Jaumandreu, 2013). In the simulation design we also let the optimal choice of labor be subject to an optimization error so that observed labor is the optimal labor choice plus a white noise error term. This adjustment addresses the collinearity problem pointed out by Ackerberg et al. (2015). The underlying parameters used to generate these datasets and a full description of the DGP are in Appendix B.

At the beginning of each period t the firm observes capital K_{jt}, productivity ω_{jt} and firm-specific input prices. As in GLZ, firms are price takers in the input markets but input prices may be different across firms and over time because of productivity or other reasons. It then optimally chooses labor and materials to maximize static profits in each period,

$$\underset{L_{jt}, M_{jt}}{Max}\, P_{jt} Q_{jt} - P_{L_{jt}} L_{jt} - P_{M_{jt}} M_{jt}$$

with demand given by

$$P_{jt} = P_t \left(\frac{Q_{jt}}{Q_t} \right)^{\frac{1}{\mu}}$$

where Q_t and P_t are industry-level output quantity and price in period t, $\mu < -1$.

In this environment, input demands at time t are functions of productivity and input prices at time t, the three exogenous drivers of the model.

We simulate a sequence of productivity, ω_{jt}, input prices, P_{Ljt} and P_{Mjt} and capital stock, K_{jt}, for each of 500 firms over 50 periods. With these variables we derive the optimal choice of labor and material inputs, investment, output quantity and output price level. That is, for each firm j and period t we generate data on $\{\omega_{jt}, K_{jt}, P_{Ljt}, P_{Mjt}, L_{jt}, M_{jt}, I_{jt}, Q_{jt}, P_{jt}\}_{t=1}^{T}$. We repeat this process 1,000 times so that we have 1,000 replications of these data. To minimize the impact of initial conditions, we keep the last $T < 50$ observations for each firm so the length of the simulated panel is T periods (the baseline T is 15 periods).

3.2 The empirical production function

The estimated production function depends on the type of price data available to the researcher. We first consider the ideal situation where (log) price levels $\{p_{Ljt}, p_{Mjt}, p_{jt}\}$ are observed by the researcher. Equivalently, the researcher observes physical quantities $\{q_{jt}, k_{jt}, l_{jt}, m_{jt}\}$. In this ideal case the empirical production function is

$$q_{jt} = \alpha_L l_{jt} + \alpha_M m_{jt} + \alpha_K k_{jt} + \omega_{jt} + \epsilon_{jt}. \tag{7}$$

The second, empirically relevant, case arises when researchers observe only output and materials price changes. In this case, the data used in the simulations are $\{r_{jt}, k_{jt}, l_{jt}, e_{jt}, p_{Ljt}, \Delta p_{Mjt}, \Delta p_{jt}\}$. In this case, the empirical production function is

$$r^*_{jt} = \alpha_L l_{jt} + \alpha_M m^*_{jt} + \alpha_K k_{jt} + \omega_{jt} + p_{jb} - \alpha_M p_{Mjb} + \epsilon_{jt}. \tag{8}$$

with partially deflated revenues and materials.[13]

[13]The base year is the first period the firm appears in the sample. Estimation starts in year $t = 1$ and uses $T < 50$ years

In the simulations results we compare three estimation results. First, those obtained when quantities are observed, equation (7). Second, results obtained when the unobserved base year price levels are normalized to zero, equation (8) with $p_{jb} = p_{Mjb} = 0$. The third type of results are also obtained from equation (8) when normalization is not imposed, i.e., treating the unobservable base year price levels as firm-specific fixed effects.

3.3 Simulation results

It is well known that a main difficulty in estimating production functions is dealing with the fact that the researcher does not observes the firm's productivity. Thus, in order to abstract from estimation issues related to the endogeneity of productivity, we will first consider the case where firm productivity is observed by the researcher. In this *thought experiment* we can estimate the production function by OLS and, hence, focus directly on the impact of normalizing the base year prices.

We then consider the case where productivity is not observed. In this case we use the Olley and Pakes (1996) proxy method when the base year price levels are normalized to zero. When this normalization is not imposed, and the unobserved base year price levels are treated as firm-specific fixed effects, we use the partially linear series estimator proposed by Baltagi and Li (2002).[14]

The estimation methods used in the paper fit the DGP chosen (and vice-versa) and are meant to illustrate the main goal of the paper: drawing attention to the bias introduced by the normalization of unobserved base year prices. The literature offers a variety of estimation methods to estimate production function under different sets of assumptions. However, the measurement error due to the normalization of unobserved base year prices is unrelated to the estimation method.

3.3.1 A thought experiment: observed productivity

In this Section we consider the hypothetical situation in which the researcher observes firm productivity. This assumption allows us to focus on the omitted price bias by eliminating the endogeneity

of data for each firm because we observe price changes for the first year in the sample.

[14] We use the estimator proposed by Baltagi and Li (2002) because of its computational simplicity compared, for example, to that of Su and Ullah (2006), which was used in the 2013 version of the paper.

problem due to unobserved productivity. That is, the empirical production function can simply be estimated by OLS and by FE-OLS (OLS augmented with a set of firm fixed effects). In Table 1 we present estimates of the empirical production function, averaged over the 1,000 replications, under alternative price data availability scenarios.

Table 1: The impact of base year price normalization with *observed* productivity

	Price *levels*	Price *changes*			
		Low elasticity		High elasticity	
	Q	I	II	III	IV
α_M	0.4000 (0.0007)	0.7023 (0.0451)	0.4000 (0.0009)	0.5036 (0.0224)	0.4000 (0.0007)
α_L	0.4000 (0.0006)	0.2305 (0.0433)	0.4000 (0.0008)	0.3021 (0.0213)	0.4000 (0.0007)
α_K	0.2000 (0.0014)	0.0705 (0.0541)	0.2000 (0.0014)	0.1194 (0.0344)	0.2001 (0.0015)
Normalization		Yes	No	Yes	No
	OLS	OLS	FE-OLS	OLS	FE-OLS

[a] Data consist of 500 firms with 15 periods each. Entries are averages over 1000 repetitions of the estimated parameters in each simulation with standard errors in parentheses.
[b] Normalization: "Yes" means that $p_{jb} = p_{Mjb} = 0$ and "No" means that the base price years are treated as firm-specific fixed effects.
[c] Low elasticity -1.05; High elasticity -4.0.
[d] "OLS" means estimation is using OLS and "FE-OLS" means it is using OLS with a set of firm-specific fixed effects.

The benchmark estimates are presented in the column labeled Q. In this case, prices are observed so that physical quantities are used in estimation, as in equation (7). These estimates are the basis for assessing the effect of the lack of full information on prices – when only price *changes* are observed – on the estimates in columns I to IV.

In Columns I and III the unobserved base year price levels are normalized to zero and the parameters are estimated by OLS. In columns II and IV the unobserved base year price levels are treated as firm-specific fixed effects and we use FE-OLS. We show results for a low and a high price elasticity of demand.

As expected, when productivity is observed OLS estimates of the quantity production function, in column Q, are equal to the true parameters (0.4 for materials and labor elasticities and 0.2 for capital elasticity). However, when base year price levels are ignored (columns I and III), the estimates are far off the true values. For example, materials elasticity increases to 0.7 or 0.5, depending on the elasticity of demand, while capital elasticity estimates decrease to 0.07 or 0.12.

This bias is a consequence of the measurement error introduced by the unobserved base year price level, $p_{jb} - \alpha_M p_{Mjb}$. When these prices are ignored – normalized to zero – a bias arises because input demands are correlated with the omitted base year prices. This correlation arises because, first, p_{Mjb} is necessarily correlated with partially deflated materials since $m^*_{jt} = m_{jt} + p_{Mjb}$. And, second, because input demands at time t depend on input prices at time t and, since these are serially correlated, labor and materials will also be correlated with p_{Mjb}.[15]

The magnitude of the bias depends on how much prices vary across firms and this variation is affected by the curvature of the demand function. Because of the presence of idiosyncratic shocks each firm is on a different supply curve which gives rise to different output prices. The variation across firms in (output) prices is inversely related to the elasticity of demand. The lower the demand elasticity, the larger the variation in output price across firms and this increases the magnitude of the omitted variable bias, as observed from the comparison between columns I and III.

Finally, treating the unobserved base year prices as firm-specific fixed effects correlated with input demands eliminates the omitted variable bias, as shown in columns II and IV.

The main point emerging from Table 1 is that, despite productivity being observed, OLS estimates are biased when the base year price levels are normalized to zero, as in columns I and III. The bias disappears when the base year prices are accounted for by firm specific fixed-effects in columns II and IV. These results deliver the main message of the paper: ignoring the unobserved base year prices

[15]Another channel for the endogeneity of the regressors is through the correlation between p_{jb} and input demands. Because output price depends on productivity and the latter is serially correlated, p_{jb} is correlated with time t productivity ω_{jt}. And, because input demands at time t depend on time t productivity, a correlation emerges between p_{jb} and input demands in other years. In this Section, however, this channel is closed because we control for productivity in the regression.

biases the estimation of the production function parameters.

3.3.2 Unobserved productivity

In this Section we return to the relevant empirical case where the researcher does not observe the firm's productivity. As mentioned above, we use the proxy method developed by Olley and Pakes (1996; OP hereafter) to estimate the production function parameters when the unobserved base year prices are normalized to zero.

The OP proxy method proceeds in two stages. In the first stage the static input parameters are identified under the assumption that productivity is the only unobservable and that it can be inverted into an observable decision variable, i.e., investment. In the second stage, the dynamic input parameters are recovered.

We present the simulation results for each stage separately. We do this for several reasons. First, there is an increasing interest in analyzing firm-level markups and first stage static inputs parameters are necessary for their estimation (e.g., Rubens, 2023; Raval, 2023; De Ridder et al., 2022; De Loecker, 2016, 2020). Second, the measurement error introduced by the unobserved base year prices affects the estimated parameters only via the first stage of the estimation procedure. Third, this implies that we can proceed in the second stage as in the usual proxy estimation methods.

Previewing our findings, the Monte Carlo results suggest that the estimated static and dynamic input elasticities are significantly biased when the unobserved base year price levels are normalized to zero. The bias, however, disappears when the measurement error arising from the unobserved base year prices is captured by firm-specific fixed effects in the first stage of OP. These results exhibit the same pattern as those in the previous Section where productivity was observed.

OP first stage: static input parameters

The OP approach relies on the notion that the investment policy rule can be inverted to proxy for unobserved productivity in the production function equation, $\omega_{jt} = h(k_{jt}, i_{jt})$, where i_{jt} represents the firm's investment. Hence, substituting for productivity in (8) the empirical production function becomes

$$r^*_{jt} = \alpha_M m^*_{jt} + \alpha_L l_{jt} + \phi(k_{jt}, i_{jt}) + p_{jb} - \alpha_M p_{Mjb} + \epsilon_{jt} \tag{9}$$

where $\phi(k_{jt}, i_{jt}) = \alpha_K k_{jt} + h(k_{jt}, i_{jt})$. In empirical applications $\phi(k_{jt}, i_{jt})$ is usually approximated by a polynomial expansion.

When the base year are normalized to zero, so that $p_{jb} - \alpha_M p_{Mjb} = 0$, the first stage corresponds to the standard OP procedure. When the base year prices are treated as firm-specific fixed effects we employ the Baltagi and Li (2002; BL hereafter) estimator. The BL estimation procedure was developed for semiparametric partial linear models with fixed effects. It involves differencing and using a series approximation to the nonparametric component of the partial linear model.[16]

Table 2 presents the averages of the estimated static input elasticities obtained in the first stage for different panel lengths. The table has the same format as that of Table 1. Column Q, the benchmark case, reports the Olley-Pakes (OP) estimates using physical quantities.

As in Table 1, when normalizing the base year prices to zero, in columns I and III, the estimates are far off the true values. The estimator of the materials elasticity is biased upwards while that of labor elasticity is biased downwards. Also as in Table 1, the biases diminish as the elasticity of demand increases.

[16] Using obvious notation, let the production function be $q_{it} = x'_{jt}\beta + \phi(z_{jt}) + \tau_j + \eta_{jt}$ for $j = 1, ..., N$ and $t = 1, ...T$. This is a semiparametric partial linear model with fixed effects. Differencing with respect to $t = t_0$ implies $\Delta q_{it} = \Delta x'_{jt}\beta + \Delta\phi(z_{jt}) + \Delta\eta_{jt}$ with $\Delta d_{jt} = d_{jt} - d_{jt_1}$. Baltagi and Li (2002) propose to approximate $\phi(z_{jt}) = p^K(z_{jt})'\theta$ where $p^K(z_{jt})$ are the first K terms $\left(p_1(z_{jt}), ..., p_K(z_{jt})\right)'$ of a sequence of functions and θ is a conformable vector of parameters

Table 2: The impact of base year price normalization with *unobserved* productivity
First stage static input parameters

	Price *levels*	Price *changes*			
		Low elasticity		High elasticity	
	Q	I	II	III	IV
		T = 7			
α_M	0.4000	0.7308	0.4000	0.5329	0.4000
	(0.0009)	(0.0478)	(0.0023)	(0.0247)	(0.0019)
α_L	0.4000	0.1601	0.4000	0.2726	0.4000
	(0.0009)	(0.0414)	(0.0023)	(0.0215)	(0.0019)
		T = 15			
α_M	0.4000	0.6853	0.4000	0.4954	0.4000
	(0.0007)	(0.0428)	(0.0016)	(0.0219)	(0.0014)
α_L	0.4000	0.2336	0.4000	0.3081	0.4000
	(0.0006)	(0.0410)	(0.0015)	(0.0209)	(0.0013)
		T = 30			
α_M	0.4000	0.6579	0.4000	0.4769	0.4000
	(0.0005)	(0.0400)	(0.0011)	(0.0210)	(0.0009)
α_L	0.4000	0.2838	0.4000	0.3285	0.4000
	(0.0005)	(0.0350)	(0.0011)	(0.0201)	(0.0009)
Normalization		Yes	No	Yes	No
		OP	BL	OP	BL

See notes to Table 1.
"OP" means estimation follows Olley-Pakes (1996) and "BL" means it follows Baltagi and Li (2002) as explained in the text.

In addition, Table 2 suggests that the bias decreases with the length T of the panel. For example, the bias of the estimator of α_M decreases from 0.33 $(0.73 - 0.40)$ to 0.26 $(0.66 - 0.40)$ as T increases from 7 to 30 periods. The bias, however, does not disappears for empirically relevant values of T.[17] This result is consistent with the following "intuitive" explanation. When the length of the panel is

[17] Although $T = 30$ is unusual in firm-level panel datasets, we use it because the length of firm-level panel datasets is increasing over time. For example, the Spanish EESE dataset now covers firms for more than 20 years, and the Bank of Italy's INVIND dataset covers firms for nearly 40 years.

short, base year b is not that "far away" from other years in the sample and the serial correlation among the observations is (relatively) strong. Conversely, in longer panels the serial correlation among the observations is weaker because observations are farther apart in time. Because the serial correlation in prices links the prices in the base year to the input demands in other years, we should expect the magnitude of the bias to decrease with the length T of the panel.

Finally, columns II and IV show that the omitted price bias disappears when the unobserved base year prices are treated as firm-specific fixed effects and the parameters are estimated by BL. Note also that the variation across simulated samples is an order of magnitude smaller for the BL estimator meaning that it delivers the correct parameters not only on average, but in many samples.

In sum, the message from Table 2 is in line with that of the *thought experiment* in Table 1: normalizing the base year prices to zero may significantly bias the estimators of the production function parameters. Moreover, this bias does not vanish for reasonable panel length sizes. From the perspective of the markup debate, these results are worrisome because they suggest that the estimated markups may be biased because of the normalization of base year prices. The bias is eliminated, however, when the unobserved base year prices are treated as firm-specific fixed effects.

OP second stage: dynamic input parameter

Once the static input parameters and the unanticipated output shock, ϵ_{jt}, are identified in the first stage, the capital coefficient α_K is recovered through orthogonality conditions based on timing assumptions. Specifically, it is assumed that capital at time t is decided at time $t-1$ and is therefore orthogonal to the productivity shock at t, i.e., $E\left[\epsilon^{\omega}_{jt}k_{jt}\right] = 0$, where $\omega_{jt} = g(\omega_{jt-1}) + \epsilon^{\omega}_{jt}$, with $g(\cdot)$ unknown. Here we follow the approach of Ackerberg et al. (2015) and De Loecker (2011) to estimate the dynamic input's parameter. This approach is based on recovering ϵ^{ω}_{jt} from $\epsilon^{\omega}_{jt} = \omega_{jt} - g(\omega_{jt-1}) = \omega_{jt} - g(\phi(i_{jt-1}, k_{jt-1}) - \alpha_K k_{jt})$, where $\phi(i_{jt}, k_{jt})$ is estimated in the first stage (see (9)) and the unknown function $g(\cdot)$ is approximated by a polynomial expansion.[18]

[18] An advantage of the Baltagi and Li (2002) estimator is that it is easy to recover the function $\phi(\cdot)$ from the first stage and then proceed to estimate the capital elasticity parameter as usually done in any proxy estimation method, e.g., OP. However, as BL suggest, the rate of convergence of the sieve estimator is slower than the parametric rate.

Table 3 presents the average, over the 1,000 replications, of the estimated capital elasticities obtained in the second stage. The table's format is the same as that in Tables 1 and 2.

Table 3: The impact of base year price normalization with *unobserved* productivity
Second stage dynamic input parameter

	Price levels observed	Price changes observed			
		Low elasticity		High elasticity	
	Q	I	II	III	IV
		T = 7			
α_K	0.1996	-1.004	0.2155	-0.3061	0.2157
	(0.2738)	(0.9959)	(0.4774)	(0.3890)	(0.4741)
		T = 15			
α_K	0.2018	-0.0853	0.2047	0.0380	0.2048
	(0.1044)	(0.1501)	(0.2096)	(0.1221)	(0.2097)
		T =30			
α_K	0.2028	0.1539	0.2078	0.1740	0.2079
	(0.0358)	(0.0380)	(0.0952)	(0.0361)	(0.0951)
Price Normalization		Yes	No	Yes	No
	OP	OP	BL	OP	BL

See notes to Table 2.

In line with our previous results, the estimates in Table 3, columns I and III, show that the dynamic input estimates are biased when the base year price level is normalized to zero. The bias can be considerable as shown in column I of the top panel. Thus, despite the second stage not being directly affected by the measurement error, the bias in the first stage carries over to the second stage. Note also that the bias decreases with T but it does not vanish for empirically relevant values of T.

The bias essentially disappears when introducing a fixed effect in the first stage to account for the unobserved base year prices, as in columns II and IV.

The variability across replications in the estimates of α_K is an order of magnitude larger that of the estimates of the static input parameters (in Table 2). This is a reflection of the inherent lower over-time variability in the capital variable, which is typical of the data used in empirical work. Because of this variability we also looked at the median of the estimates of α_K (not shown). In columns II and IV, the median values of the estimates of α_K are closer to the true values than the mean value meaning that there is a similar number of estimates falling above and below the true value.

In sum, the goal of the Monte Carlo simulations was to quantify the bias introduced when base year prices are normalized to zero. Using parameters taken from the literature, and summarized in Appendix Table B1, the results point to significant biases in the estimation of the production function parameters. In Appendix C we find this conclusion to be robust to changes in some of the parameters generating the simulated data.

There is an additional problem when the base year price level is normalized to zero: the production function parameter estimates may not invariant to the choice of base year. This is the topic of next Section.

4 Estimates are sensitive to the choice of base year

The choice of base year should not affect the estimated parameters of the production function when quantities are used because equation (7)) does not depend on prices. Similarly, if prices are observed and nominal variables are fully deflated, the choice of base year should also have no effect on the estimates.[19]

The question we want to answer in this Section is whether the production function parameter estimates are sensitive to the choice of base year when the unobserved base year price levels are normalized to zero, i.e., when partially deflated variables are used. This is an important question because the base year used in the recursion formula (4) is chosen by the researcher and can be any

[19]Output price, for example, can be written as $p_{jt} = p_{jb} + (p_{jt} - p_{jb}) = p_{jb'} + (p_{jt} - p_{jb'})$ where b and b' are two different base years. Changing the base year does not affect the price and, therefore, should not affect the estimates. The same holds for the materials price.

year in the sample.

At first glance, the empirical production function depends on the choice of base year through the unobserved base year price levels (see equation (8)). Choosing different base years implies normalizing the price level to zero at different points in time, i.e., at year b or at year b'. Thus, the same nominal variables (revenues and expenditures) are deflated differently reflecting the price change between year t and the base years b or b'.[20] This implies that a change in the base year actually changes the variables used in the regression and will, therefore, affect the parameter estimates.

This sensitivity to the choice of base year does not occur when incorporating firm-specific fixed effects into the estimation because first-differenced variables do not depend on the base year.[21]

We use the previous Monte Carlo simulation design to illustrate this point. We estimate the production function under two base years and construct the ratio of the estimates. If the choice of base year does not matter then this ratio should be unity.

We focus on the empirically relevant case of unobserved productivity. The results in Table 2 were based on choosing the first year of the sample as base year, $b = 1$. We now re-estimate the elasticities, using the same simulated data underlying the results in Table 2, but with a base year in the middle of the sample, $b' = 7$ (the length of the panel is 15 periods). Table 4 presents the average ratios over the 1,000 data replications.

[20] In the case of output, for example, when the base year is b, we set $p_{jb} = 0$, and the output price used to construct partially deflated revenues is $(p_{jt} - p_{jb})$, for $t > b$, the price change between t and b. Similarly, when the base year is b' and we set $p_{jb'} = 0$, the price deflator is $(p_{jt} - p_{jb'})$, for $t > b'$. But note that, when prices differ across the base years, $(p_{jt} - p_{jb}) \neq (p_{jt} - p_{jb'})$, for $t > max(b', b)$

[21] When estimating model (9) accounting for the fixed effects we are estimating the model with first-differenced variables. We now show that although r^*_{jt} and m^*_{jt} depend on the choice of base year, their first-differences do not. We have $r^*_{jt} = r_{jt} - \sum_{s=b+1}^{t} \Delta p_{js} = q_{jt} + p_{jb}$ making it clear that, in general, partially deflated revenues depend on the choice of base year. Taking first-differences we get $r^*_{jt} - r^*_{jt-1} = r_{jt} - r_{jt-1} - \Delta p_{jt} = q_{jt} - q_{jt-1}$ which does not depend on the choice of base year because physical output does not depend on it. Similarly for m^*_{jt}.

Table 4: Ratio of estimates in two different base years.

	Price levels	Price changes			
		Low elasticity		High elasticity	
	Q	I	II	III	IV
α_M	1.00	1.075	1.00	1.069	1.00
	(0.00)	(0.090)	(0.00)	0.063)	(0.00)
α_L	1.00	0.778	1.00	0.906	1.00
	(0.00)	(0.219)	(0.00)	(0.092)	(0.00)
α_K	1.00	0.5335	1.00	0.7770	1.00
	(0.00)	(0.8633)	(0.00)	(0.9277)	(0.00)
Normalization		Yes	No	Yes	No
	OP	OP	BL	OP	BL

See notes to Table 2. T=15.
The entries are the average of the ratio of the estimates obtained when changing the base year from the first to the seventh year in the sample period.

A value different from 1 means that the estimates change when the base year is changed. Clearly, in columns I and III, where the base year price levels are normalized to zero, estimates are not invariant to the choice of base year. The differences can be significant. For example, the capital coefficient declines, on average, by 47 percent when the base year is changed (column I). Intuitively, because of its bias, the OP estimator converges to a probability limit that depends on the covariances between the inputs in years t and the prices in the base year b. If these covariances change with the choice of base year, the probability limit will also change and the estimates will differ (see Appendix A for the analysis of a simple case).

On the other hand, in Columns II and IV, where the base year prices are accounted for by firm-specific fixed effects, the estimates do not change when the base year changes. This reflects the previous observation that first-differenced variables do not depend on the choice of base year.

We remark that the choice of base year does not affect the estimates when the estimator is consistent. On the other hand, finding that the estimates do not change when the base year changes does not necessarily imply that the estimator is consistent. It simply means that the magnitude of the bias does not change with the base year (see Appendix A for the analysis of a simple case).

We also remark that studying whether this ratio is equal to or different from 1 is not meant as a statistical test of a null hypothesis of ”no effect” of the choice of base year. By definition, changing the base year should not affect the estimates when using physical quantities or fully deflated revenues and expenditures. Any deviation between the estimates implies that the estimation of the production parameters based on partially deflated nominal variables suffers from an omitted price bias.

In the next Section we present additional evidence, based on real data, on the sensitivity of the estimated coefficients to the choice of base year when normalization of the base year prices is imposed.

4.1 Changing the base year: ESEE firm-level data

We now illustrate the lack of invariance of the estimated production function parameters to the choice of base year using *real data*. We use data from the Spanish firm-level database Encuesta Sobre Estrategias Empresariales (ESEE) which has been widely used to estimate production functions and related issues. Specifically, we use the synthetic dataset generated by Koch et al. (2021), based on the ESEE, that makes replicating our results possible.[22]

The ESEE is a firm-level dataset that reports data on annual price changes. These price change data are used to recover the unobserved firm's price level by normalizing the log base year price level to zero (see, e.g.,Koch et al., 2021; Jaumandreu and Lin, 2018). We follow Koch et al. (2021) and use their data and script to estimate the labor coefficient of a Cobb-Douglas value-added production function under three choices of base year.[23]

[22]Most firm-level datasets, including the ESEE, are proprietary data but Koch et al. (2021) constructed a synthetic dataset based on the ESEE that allows for replication of results and is available at https://academic.oup.com/ej/article/131/638/2553/6124631supplementary-data.

[23]Their empirical production function is $y_{jt} = \alpha_L l_{jt} + \alpha_K k_{jt} + \omega_{jt} + p_{jb} - p_{Mjb} + \epsilon_{jt}$ where y_{jt} is value added measured by $y^* = r^* - m^*$, the difference between partially deflated revenues and materials. We focus on the labor coefficient only in order not to clutter the table.

Table 5: EESE: ratio of estimates of α_L when changing the base year

	ACF			OP			BL		
Base year	1995	2000	2015	1995	2000	2015	1995	2000	2015
Sector									
1	1.054	1.180	0.695	1.104	1.033	0.947	1.00	1.00	1.00
2	0.534	0.817	0.823	0.260	0.370	0.535	1.00	1.00	1.00
3	1.974	1.121	1.147	1.066	1.052	1.043	1.00	1.00	1.00
4	1.551	1.312	0.406	1.142	1.116	1.064	1.00	1.00	1.00
6	1.037	1.032	0.075	1.142	1.072	0.943	1.00	1.00	1.00
7	1.276	1.058	1.196	0.728	0.944	0.623	1.00	1.00	1.00
8	1.038	1.031	0.961	1.671	1.222	1.897	1.00	1.00	1.00
9	0.662	0.833	1.239	1.067	1.062	0.755	1.00	1.00	1.00
10	1.031	0.958	0.916	0.668	0.884	1.108	1.00	1.00	1.00

The table presents the average ratio of the labor coefficient estimate, over the 1,000 replications, when the base year is 2006 to that estimate when the base year is 1995, 2000 and 2015.
The dataset and scripts are those from Koch et al. (2021).
"ACF" means estimation is using Ackerberg et al. (2015), "OP" means it is using Olley-Pakes (1996) and "BL" means it is using Baltagi and Li (2002) as explained in the text.

Table 5 presents the average ratio of the labor coefficient estimate when the base year is 2006 —the base year used by Koch et al . (2021) —to that estimate when the base year is 1995 (column I), 2000 (column II) and 2015 (column III). In the first (left) panel the estimates are obtained following the Ackerberg et al. (2015) approach to estimate production functions, which is the one followed by Koch et al. (2021). In the second (central) panel we apply Olley and pakes (1996) and in the third (right) panel we apply Baltagi and Li (2002). Recall that when the ratio differs from 1 it is indicative that the estimates change when the base year changes.

The results very clearly indicate that ignoring the unobserved base year prices makes the estimates sensitive to the choice of base year. The estimated labor coefficients, based on exactly the same data, can differ by more than 50 percent when the base year is changed. This is worrisome since the choice of base year is usually viewed as an innocuous choice.

The results in this section are relevant to the "markup debate": we can change a markup estimate by simply choosing a different base year in the computation of price levels when we normalize the base year prices in the empirical production function.

5 Unobserved base year prices and productivity

In this Section we show that the usual approach to recover productivity as a residual includes the measurement error introduced by the unobserved base year prices (De Loecker, 2011).

Firm-level productivity is often measured by the residual obtained after estimation of the production function in equation (8),

$$\begin{aligned}\hat{\omega}_{jt} &= r^*_{jt} - \left(\hat{\alpha}_L l_{jt} + \hat{\alpha}_M m^*_{jt} + \hat{\alpha}_K k_{jt} + \hat{\epsilon}_{jt}\right) \\ &= q_{jt} - \left(\hat{\alpha}_L l_{jt} + \hat{\alpha}_M m_{jt} + \hat{\alpha}_K k_{jt} + \hat{\epsilon}_{jt}\right) + p_{jb} - \hat{\alpha}_M p_{Mjb} \\ &= \omega_{jt} + (\alpha_L - \hat{\alpha}_L) l_{jt} + (\alpha_M - \hat{\alpha}_M) m_{jt} + (\alpha_K - \hat{\alpha}_K) k_{jt} + (\epsilon_{jt} - \hat{\epsilon}_{jt}) + p_{jb} - \hat{\alpha}_M p_{Mjb}\end{aligned} \tag{10}$$

where $\hat{\epsilon}_{jt}$ is estimated in a first stage (e.g., as in Ackerberg et al., 2015).

When using a consistent estimator of the production function parameters (e.g., the BL estimator used in this paper), so that estimation errors vanish in large samples, we obtain that residual productivity is

$$\hat{\omega}_{jt} \approx \omega_{jt} + p_{jb} - \alpha_M p_{Mjb}. \tag{11}$$

In what follows we focus on this case and base our analysis on this approximation. The expression above shows that residual productivity includes a price effect reflected by the unobserved base year prices. Differences in $\hat{\omega}_{jt}$ across firms may reflect differences in demand conditions rather than in productivity (e.g., Foster et al., 2008). In addition, the variance of estimated productivity includes the variance of the unobserved base year prices. This much is a well-known feature of estimating production functions when quantities are not observed.

What is less known is that residual productivity depends on the choice of base year, as long as nominal prices vary over time,

$$\hat{\omega}_{jt}^{b} - \hat{\omega}_{jt}^{b'} \approx \left(p_{jb} - p_{jb'}\right) - \alpha_M \left(p_{Mjb} - p_{Mjb'}\right) \neq 0$$

where the superscripts in the estimated productivity indicate the base year chosen to construct the firm-level prices used in the deflation of nominal quantities. This implies that a simple change in the base year may lead to different conclusions regarding productivity features.

It is clear that because $p_{jb} - \alpha_M p_{Mjb}$ is constant over time, productivity *growth* at the firm level can be consistently estimated by $\hat{\omega}_{jt} - \hat{\omega}_{jt-1}$. Despite this, aggregate productivity growth might be biased. Estimated aggregate productivity growth $\hat{\Phi}_{jt}$ can be written as

$$\begin{aligned} \hat{\Phi}_{jt} - \hat{\Phi}_{jt-1} &= \sum_j s_{jt}\hat{\omega}_{jt} - \sum_j s_{jt-1}\hat{\omega}_{jt-1} \\ &\approx \sum_j s_{jt}\omega_{jt} - \sum_j s_{jt-1}\omega_{jt-1} + \sum_j (s_{jt} - s_{jt-1})(p_{jb} - \alpha_M p_{Mjb}) \end{aligned} \tag{12}$$

using (11) and where $s_{jt} = R_{jt} / \sum_j R_{jt}$ is firm $j's$ share of revenue in year t.

The last term in equation (12) introduces a bias in estimated aggregate productivity. It reflects the covariance between changes in market shares and prices in the base year. If more productive firms have lower prices and increasing market shares we would expect this term to be negative so that $\hat{\Phi}_{jt} - \hat{\Phi}_{jt-1}$ would be underestimating aggregate productivity growth. Note also that the magnitude of the bias depends on the units in which prices are measured so that its value can be increased or decreased arbitrarily.

In sum, using residual based productivity estimates gives biased firm-specific productivity levels which also depend on the choice of base year, even if the production function is estimated consistently. Although we can recover firm-specific productivity growth, aggregate productivity growth is still biased and this bias is likely to be negative.

Instead of using residual productivity estimates we can estimate firm-specific productivity levels directly from the second stage, as suggested by De Loecker (2011), which is free of unobserved base year prices. To compare these two estimates we use the same simulation design underlying the results in Table 2 to compute a "residual productivity" given by $\hat{\omega}_{jt} = \omega_{jt} + p_{jb} - \alpha_M p_{Mjb}$, and a "second stage productivity" given by $\hat{\omega}_{jt} = \hat{\phi}(i_{jt}, k_{jt}) - \alpha_K k_{jt}$, where $\hat{\phi}(i_{jt}, k_{jt})$ is estimated from the BL first stage. Note that these measures are not truly productivity estimates since we are using the unobserved base prices and the true capital elasticity in the computation of these measures. Doing this avoids estimation errors and allows for a cleaner comparison between the two approaches.

Table 6 compares the two productivity measures to the true productivity. The standard deviation of true productivity is 0.26 which, as expected, is overestimated when using residual productivity (0.35). On the other hand, second stage productivity stage delivers the correct dispersion. The column labeled β_{OLS} shows the OLS coefficient estimate from a regression of ω_{jt} on $\hat{\omega}_{jt}$. We clearly see that we can recover the true productivity when using the second stage productivity but not the residual one.

Table 6: Productivity and unobserved base year price level.

		Low elasticity		High elasticity	
Productivity		std. dev. productivity	β_{OLS}	std. dev. productivity	β_{OLS}
True	ω_{jt}	0.2599 (0.0027)		0.2599 (0.0027)	
Residual	$\hat{\omega}_{jt} = \omega_{jt} + p_{jb} - \alpha_M p_{Mjb}$	0.3535 (0.0074)	0.4485 (0.0183)	0.2891 (0.0045)	0.7245 (0.0152)
Second Stage	$\hat{\omega}_{jt} = \hat{\phi}(i_{jt}, k_{jt}) - \alpha_K k_{jt}$	0.2605 (0.0029)	0.9960 (0.0058)	0.2608 0.0029)	0.9960 (0.0060)

[a] Data consist of 500 firms with 15 periods each. Entries are averages over 1000 repetitions of the estimated parameters in each simulation with standard errors in parentheses.
[b] Low elasticity -1.05; High elasticity -4.0.

6 Conclusion

The estimation of production functions with firm-level data is hampered by the non-observability of physical quantities (output and inputs). Recent studies that use revenues and input expenditures deflated by industry-level prices introduce structural assumptions to cope with the omitted variable bias identified by Klette and Griliches (1996).

As data on firm-level price changes become increasingly available, researchers use them to construct firm-specific prices which are then used to deflate nominal revenues and/or input expenditures. The goal of this paper is to show that this procedure introduces a measurement error reflecting the base year prices used in the construction of the firm-specific prices.

This measurement error generates an omitted variable bias similar in spirit to that of Klette and Griliches (1996). Using Monte Carlo simulations we find that ignoring this measurement error – by its normalization to zero – biases the estimated parameters of the production function.

Another consequence of the normalization of the base year prices is that the estimates of the production function are not invariant to the choice of base year, under the usual assumptions. This implies, for example, that markup estimates based on estimated elasticities of the static inputs may also depend on the choice of base year. Moreover, as shown in the paper, residual productivity estimates also vary with the choice of base year, even in cases when the production function parameters are estimated consistently.

Finally, we also show that the bias introduced by the measurement error may be addressed by treating the base year prices as firm-specific fixed effects in the estimation of the production function. The unobserved *output* price always enters additively into the empirical production function and is therefore amenable to "fixed effect estimation". The way the unobserved *input* price enters the regression model depends on the functional form of the production function. In our Cobb Douglas example, it also enters additively and can be easily captured by firm-specific fixed effects. In non-linear production models, this may require more complex estimation procedures.

References

Ackerberg, D., Benkard, C. L., Berry, S., and Pakes, A. (2007). Econometric tools for analyzing market outcomes. *Handbook of Econometrics*, 6:4171–4276.

Ackerberg, D. A., Caves, K., and Frazer, G. (2015). Identification properties of recent production function estimators. *Econometrica*, 83(6):2411–2451.

Baltagi, B. and Li, D. (2002). Series estimation of partially linear panel data models with fixed effects. *Annals of economics and finance*, 3(1):103–116.

Blundell, R. and Bond, S. (2000). Gmm estimation with persistent panel data: an application to production functions. *Econometric reviews*, 19(3):321–340.

Carlsson, M., Messina, J., and Nordstrom Skans, O. (2021). Firm-level shocks and labour flows. *The Economic Journal*, 131(634):598–623.

Chen, C. and Steinwender, C. (2021). Import competition, heterogeneous preferences of managers, and productivity. *Journal of International Economics*, 133:103533.

De Loecker, J. (2011). Product differentiation, multiproduct firms, and estimating the impact of trade liberalization on productivity. *Econometrica*, 79(5):1407–1451.

De Loecker, J., Goldberg, P. K., Khandelwal, A. K., and Pavcnik, N. (2016). Prices, markups, and trade reform. *Econometrica*, 84(2):445–510.

De Loecker, J. and Syverson, C. (2021). An industrial organization perspective on productivity. In *Handbook of industrial organization*, volume 4, pages 141–223.

De Ridder, M., Grassi, B., Morzenti, G., et al. (2022). The hitchhiker's guide to markup estimation.

Dolado, J. J., Ortigueira, S., and Stucchi, R. (2016). Does dual employment protection affect tfp? evidence from spanish manufacturing firms. *SERIEs*, 7:421–459.

Doraszelski, U. and Jaumandreu, J. (2013). R d and productivity: Estimating endogenous productivity. *Review of economic studies*, 80(4):1338–1383.

Eslava, M., Haltiwanger, J., Kugler, A., and Kugler, M. (2004). The effects of structural reforms on productivity and profitability enhancing reallocation: evidence from colombia. *Journal of development Economics*, 75(2):333–371.

Foster, L., Haltiwanger, J., and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review*, 98(1):394–425.

Gandhi, A., Navarro, S., and Rivers, D. A. (2020). On the identification of gross output production functions. *Journal of Political Economy*, 128(8):2973–3016.

Grieco, P. L., Li, S., and Zhang, H. (2016). Production function estimation with unobserved input price dispersion. *International Economic Review*, 57(2):665–690.

Jaumandreu, J. and Lin, S. (2018). Prices under innovation: evidence from manufacturing firms.

Klette, T. J. and Griliches, Z. (1996). The inconsistency of common scale estimators when output prices are unobserved and endogenous. *Journal of applied econometrics*, 11(4):343–361.

Koch, M., Manuylov, I., and Smolka, M. (2021). Robots and firms. *The Economic Journal*, 131(638):2553–2584.

Levinsohn, J. and Petrin, A. (2003). Estimating production functions using inputs to control for unobservables. *The review of economic studies*, 70(2):317–341.

Mairesse, J. and Jaumandreu, J. (2005). Panel-data estimates of the production function and the revenue function: what difference does it make? *The Scandinavian Journal of Economics*, 107(4):651–672.

Marschak, J. and Andrews, W. H. (1944). Random simultaneous equations and the theory of production. *Econometrica*, 12(3):143–205.

Mertens, M. (2020). Labor market power and the distorting effects of international trade. *International Journal of Industrial Organization*, 68:102–562.

Morlacco, M. (2020). Market power in input markets: Theory and evidence from french manufacturing. *Unpublished*.

Olley, S. and pakes, A. (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, 64(6):1263–1297.

Pakes, A. (2021). A helicopter tour of some underlying issues in empirical industrial organization. *Annual Review of Economics*, 13:397–421.

Raval, D. (2023). Testing the production approach to markup estimation. *Review of Economic Studies*, 90(5):2592–2611.

Rubens, M. (2023). Market structure, oligopsony power, and productivity. *American Economic Review*, 113(9):2382–2410.

Smeets, V. and Warzynski, F. (2013). Estimating productivity with multi-product firms, pricing heterogeneity and the role of international trade. *Journal of International Economics*, 90(2):237–244.

Su, L. and Ullah, A. (2006). Profile likelihood estimation of partially linear panel data models with fixed effects. *Economics Letters*, 92(1):75–81.

Syverson, C. (2011). What determines productivity? *Journal of Economic literature*, 49(2):326–365.

Appendix A The bias in the OLS estimator

In this Appendix we use the simple regression model to examine the conditions under which the zero normalization of base year prices results in (a) biased estimators, and in (b) estimators sensitive to the choice of base year.

We assume a single input, materials, and a linear production function, $q_{jt} = \alpha_0 + \alpha_M m_{jt} + \epsilon_{jt}$. To focus exclusively on the impact of normalizing the base year prices we assume that productivity is observed and that q_{jt} measures output net of productivity. As explained in the text, when only price changes are observed we use partially deflated revenues and materials to estimate α_M. Equation (6) in the text becomes

$$\begin{aligned} r^*_{jt} &= \alpha_M m^*_{jt} - \alpha_M p_{jb} + p_{jb} + \epsilon_{jt} \\ &= \alpha_M m^*_{jt} + v_{jb} + \epsilon_{jt} \end{aligned} \tag{A.1}$$

where $v_{jb} = p_{jb} - \alpha_M p_{Mjb}$, $r^*_{jt} \equiv r_{jt} - \sum_{s=b+1}^{t} \Delta p_{js}$ and $m^*_{jt} = e_{jt} - \sum_{s=b+1}^{t} \Delta p_{Mjs} = m_{jt} + p_{Mjb}$ as defined in the text.

The standard approach in the literature has been to ignore the unobserved base year prices by normalizing them to zero. To understand under which conditions this normalization leads to biased estimators we derive the probability limit (plim) of the OLS estimator of α_M in (A.1).

We assume that we have a panel of N firms over T periods. The firms are identically and independently distributed. Materials demand and prices within a firm can be serially correlated but they are independent across firms. In this single-regressor model we can use the well known OLS formula to obtain,

$$plim(\hat{\alpha}_M) = \alpha_M - \alpha_M \frac{\sum_{t=1}^{T} Cov(m^*_{jt}, v_{jb})}{\sum_{t=1}^{T} Var(m^*_{jt}) + \sum_{t=1}^{T} E(m^*_{jt}) \left[E(m^*_{jt}) - \frac{1}{T} \sum_{t=1}^{T} E(m^*_{jt}) \right]} \tag{A.2}$$

as $N \to \infty$.[24]

[24]This result relies on the assumption that partially deflated materials are uncorrelated with the structural error ϵ_{jt} in all periods (strict exogeneity).

The first and second moments in (A.2) may vary over time t but are the same across firms because of the i.i.d sampling assumption (the j subscript can be ignored). To be clear, the moments in (A.2) refer to the moments in the common population of firms at time t, i.e., $Var(m^*_{jt})$ is the population variance across firms at time t.

For the OLS estimator to be consistent we need $Cov(m^*_{jt}, v_{jb}) = 0$ for all t. By definition of the partially deflated materials, however, it will almost always be the case that these covariances are nonzero because $m^*_{jt} = m_{jt} + p_{Mjb}$. This holds irrespective of the serial correlation in prices and their effect on materials demand. The exception is when there is no variation in prices across firms so that v_{jb} is a constant. Furthermore, in most economic environments theory tells us that materials demand is a function of materials and output prices and this may additionally imply a nonzero $Cov(m^*_{jt}, v_{jb})$ term.

To make further progress we impose some structure on the input demands. We assume that firms follow a model of monopolistic competition with inverse demand,[25].

$$P_{jt} = Q_{jt}^{\frac{1}{\mu}}, \qquad \mu < -1$$

Firms maximize static profits by choosing output price and the amount of materials, given materials prices. In this model (log) materials demand and log output price are linear functions of the exogenous log materials price p_{Mjt} denoted by

$$\begin{aligned} m_{jt} &= \gamma_0 + \gamma_1 p_{Mjt} \\ p_{jt} &= \delta_0 + \delta_1 p_{Mjt} \end{aligned} \tag{A.3}$$

where the parameters are functions of μ and α_M.[26]

This implies that $v_{jb} = p_{jb} - \alpha_M p_{jb} = \delta_0 + \lambda p_{Mjb}$, with $\lambda = \delta_1 - \alpha_M$. Recall that $m^*_{jt} = m_{jt} + p_{Mjb}$. We then have

[25] This model is a simplified version of the model used to generate data for our Monte Carlo simulations (see Section 3 and Appendix B for details)

[26] For example, $\gamma_1 = -\frac{\mu}{\mu - \alpha_M(1+\mu)} < 0$ and $\delta_1 = \frac{\alpha_M}{\mu}\gamma_1 > 0$.

$$\begin{aligned} Cov(m^*_{jt}, v_{jb}) &= Cov(m_{jt} + p_{Mjb}, \lambda p_{Mjb}) \\ &= \lambda\gamma_1 Cov(p_{Mjt}, p_{Mjb}) + \lambda Var(p_{Mjb}) \end{aligned}$$

Plugging back these expressions into (A.2) we get that for any choice of base year b,

$$plim(\hat{\alpha}_M) = \alpha_M - \alpha_M \frac{T\lambda Var(p_{Mjb}) + \sum_{t=1}^{T} \lambda\gamma_1 Cov(p_{Mjt}, p_{Mjb})}{\sum_{t=1}^{T} Var(m^*_{jt}) + \sum_{t=1}^{T} E(m^*_{jt}) \left[E(m^*_{jt}) - \frac{1}{T} \sum_{t=1}^{T} E(m^*_{jt}) \right]} \tag{A.4}$$

We can use formula (A.4) to derive conditions under which ignoring the unobserved base year prices leads to consistent OLS estimators. The key moments are the variance of prices across firms in the base year b and the covariances between period t and period b prices (also across firms).

In general, the OLS estimator is not consistent because materials prices usually vary across firms (in the base year), $Var(p_{Mjb}) \neq 0$, and they are usually serially correlated over time, $Cov(p_{Mjt}, p_{Mjb}) \neq 0$ for some t. This implies $plim(\hat{\alpha}_M) \neq \alpha_M$.

There are however, special cases when OLS delivers a consistent estimator. This occurs when $Var(p_{Mjb}) = 0$, i.e., prices are the same across firms in the base year (although prices may differ in other years). In this case, $p_{Mjb} = p_{Mb}$ is the same for every firm and the term is absorbed into the overall regression constant; there is no omitted variable. The covariance terms in (A.4) also vanish because prices in period b do not vary across firms.

Suppose materials prices are serially uncorrelated but that prices differ in the base year. In this case, there is no transmission of the base year prices to period t materials demand but the bias in OLS does not disappear.The reason is that, by construction, partially deflated materials, $m^*_{jt} = m_{jt} + p_{Mjb}$ depend on the base year price which is part of the error. That is, even if prices vary randomly over time, OLS is still biased as long as there is variation across firms in the price of materials.

Sensitivity to the choice of base year

Another topic of interest, developed in Section 4 of the paper, is the non-invariance of the estimator to the choice of base year. The numerator of the probability limit in (A.4) helps us to understands when this happens. In general, if the estimator is consistent then it cannot change with the base year. So we focus on situations when it is not consistent.

The point is that the variance and covariances in (A.4) can vary with the choice of base year b. Intuitively, the magnitude of the OLS bias depends on the covariances between year t and year b prices, and on the variance $Var(p_{Mjb})$. If changing the base year changes these moments , the plim of the OLS estimator will change. It is only in the rare case where $Var(p_{Mjb})$ and $Cov(p_{Mjt}, p_{Mjb})$ are the same for all b, that the estimator will be invariant to the choice of base year.

Appendix B The data generating process (DGP)

In this Appendix we describe the data-generating process used to simulate data our Monte Carlo experiments. For the most part, we follow Appendix A.2 in Grieco et al. (2016; GLZ, hereafter). We simulate a dataset for 500 firms observed during 15 periods. We generate 1,000 replications of this dataset.

We first describe the generation of the exogenous variables: productivity, input prices and capital. Given these variables, we then derive the optimal choice of static inputs (labor and materials) as well as output quantity and price.

The evolution of productivity for each firm j is a first-order Markov process:

$$\omega_{jt} = \lambda_0 + \lambda_1 \omega_{jt-1} + \epsilon^{\omega}_{jt}$$

and the idiosyncratic labor and material (log) input prices are generated by

$$p_{L_{jt}} = \lambda_{L_0} + \lambda_{L_1} p_{L_{jt-1}} + \epsilon^{L}_{jt+1}$$
$$p_{M_{jt}} = \lambda_{M_0} + \lambda_{M_1} p_{M_{jt-1}} + \epsilon^{M}_{jt+1}$$

where $\left(\epsilon^L_{jt}, \epsilon^M_{jt}, \epsilon^{\omega}_{jt}\right)' \sim N(0, \Sigma)$, where Σ is a 3×3 positive matrix. When $\Sigma = \text{diag}\left(\sigma^2_L, \sigma^2_M, \sigma^2_{\omega}\right)'$ input prices are said to be exogenous. When input prices shocks are correlated with the productivity shock we say that input prices are endogenous.[27]

The investment rule and capital evolution process are set as

$$log(I_{jt}) = \xi\omega_{jt} + (1-\xi)log(K_{jt})$$
$$K_{jt+1} = K_{jt} + I_{jt}$$

where $0 < \xi < 1$. The investment rule satisfies Olley and Pakes (1996) conditions.

Using these equations, and choosing parameters and initial values, we simulate data for $\left\{\omega_{jt}, K_{jt}, P_{L_{jt}}, P_{M_{jt}}\right\}$ for each firm $j = 1, ..., 500$ and period t.

The GZL model assumes firms are monopolistically competitive and face the inverse demand function

$$P_{jt} = P_t\left(\frac{Q_{jt}}{Q_t}\right)^{\frac{1}{\mu}}$$

where Q_t and P_t are industry-level output quantity and price in period t and $\mu < -1$. As in GLZ we normalize the industry-level output quantity and price to unity.

At the beginning of each period t the firm observes K_{jt}, productivity ω_{jt} and firm-specific input prices. It then optimally chooses labor and materials to maximize static profits in each period,

$$\underset{L_{jt},M_{jt}}{Max}\, P_{jt}Q_{jt} - P_{L_{jt}}L_{jt} - P_{M_{jt}}M_{jt}$$

Departing from GLZ we assume a Cobb-Douglas production function

$$Q_{jt} = e^{\omega_{jt}}K_{jt}^{\alpha_K}M_{jt}^{\alpha_M}L_{jt}^{\alpha_L}.$$

[27] Several papers document strong persistence in output prices at the firm level (Roberts and Supina (2000), Foster et al (2008)). Foster et al. (2008), for example, find that the implied annual autocorrelation in output prices, as well as in productivity, is roughly 0.75 to 0.80.

Labor and materials satisfy the first order conditions given by

$$cQ^c_{jt}\alpha_L = P_{L_{jt}} L_{jt}$$
$$cQ^c_{jt}\alpha_M = P_{M_{jt}} M_{jt}$$

where $c = \frac{\mu+1}{\mu} P_t Q_t^{-\frac{1}{\mu}} = \frac{\mu+1}{\mu}$ under the assumption that $P_t = Q_t = 1$.

Departing from GZL we assume that observed labor demand suffers from an optimization error. Labor demand is then

$$L_{jt} = \frac{\alpha_L P_{M_{jt}}}{\alpha_M P_{L_{jt}}} M_{jt} e^{\nu_{jt}} \tag{B.1}$$

where $\nu_L \sim N(0, \sigma_{\nu_L})$ is the firm's labor demand optimization error.

Plugging (B.1) into the production function, and solving for materials gives

$$M_{jt} = \left(Q_{jt} e^{-\omega_{jt}} K_{jt}^{-\alpha_K} \left(\frac{\alpha_L P_{M_{jt}}}{\alpha_M P_{L_{jt}}} e^{\nu_{jt}} \right)^{-\alpha_L} \right)^{\frac{1}{\alpha_L+\alpha_M}} \tag{B.2}$$

Finally, output is derived from adding the two first order conditions

$$cQ^c_{jt}(\alpha_L + \alpha_M) = \left(\frac{\alpha_L}{\alpha_M} e^{\nu_{jt}} + P_{M_{jt}} \right) M_{jt}$$

substituting for materials and solving for Q_{jt},

$$Q^*_{jt} = \left[\frac{1}{c(\alpha_L + \alpha_M)} \left(\frac{\alpha_L P_{Mjt}}{\alpha_M} e^{\nu_{jt}} + P_{Mjt} \right) \left(e^{-\omega_{jt}} K_{jt}^{-\alpha_K} \left(\frac{\alpha_L P_{M_{jt}}}{\alpha_M P_{L_{jt}}} e^{\nu_{jt}} \right)^{-\alpha_L} \right)^{\frac{1}{\alpha_L+\alpha_M}} \right]^{\frac{1}{c - \frac{1}{\alpha_L+\alpha_M}}} \tag{B.3}$$

We use (B.3) to compute Q^*_{jt}, we then get materials from (B.2) using $Q_{jt} = Q^*_{jt}$ and finally labor from (B.1).

Finally, as in GZL, observed output, prices and revenues are

$$
\begin{aligned}
Q_{jt} &= Q^*_{jt} e^{\epsilon_{jt}} \\
P_{jt} &= Q_{jt}^{\frac{1}{\mu}} \\
R_{jt} &= P_{jt} Q_{jt}
\end{aligned}
$$

where $\epsilon_{jt} \sim N(0, 0.01)$ is a mesurement error.

In this way, we generate the vector of data $\left(\omega_{jt}, K_{jt}, P_{L_{jt}}, P_{M_{jt}}, M_{jt}, L_{jt}, I_{jt}, Q_{jt}, P_{jt}, R_{jt}\right)$ for each firm j and period t. For each firm j in each replication, we generate 50 observations and keep the last $T < 50$ so that the length of the panel data is T (7,15,30). This minimizes the effect of the initial conditions for the prices, productivity and capital processes. Finally, Table B1 presents the values of the parameters used in the simulations.

.

Table B1: Monte Carlo parameter values

Parameter	Description	Value	Reference
μ	Demand elasticity	Low: -1.05	GLZ^a
		High: -4.0	
α_M	Materials elasticity	0.40	GLZ
α_L	Labor elasticity	0.40	GLZ
α_K	Capital elasticity	0.20	GLZ
ξ	Parameter investment rule	0.20	GLZ
λ^b	Autocorrelation parameter	0.80	FHS^c
sd(ω)	Standard of deviation productivity	0.26	FHS
sd(price)	Standard deviation of input price levels	0.18	FHS
sd(K_0)	Standard deviation of initial capital stock	0.05	GLZ
sd(w_0)	Standard deviation of initial productivity	0.05	GLZ
sd(η)	Standard deviation of production measurement error	0.01	GLZ
sd(ν_L)	Standard deviation of labor measurement error	0.01	–
$corr(p_L, \omega)^d$	Correlation between shocks to price of labor and productivity	-0.25	FHS
$corr(p_M, \omega)^d$	Correlation between shocks to price of materials and productivity	-0.25	FHS
$corr(p_M, p_L)^d$	Correlation between shocks to prices of labor and of materials	0.25	
T	Number of periods	15	
J	Number of firms	500	
N	Number of replications	1000	

[a] Grieco et al. (2016).

[b] Autocorrelation coefficient for productivity, price of labor and price of materials. It is set to zero for labor and materials when there is no price persistence.

[c] Foster et al. (2008).

[d] Set to half of that the FHS value. Correlations are set to zero when input prices are exogenous.

Appendix C Additional simulation results

The simulation results presented in the previous section were based on data simulated assuming parameter values for drawn from the literature, as detailed in Table B1 of Appendix B. In this Appendix we summarize the results from changing some of the underlying parameters generating the simulated data. In Table C1 we present the OP and Baltagi-Li estimators, appearing in columns I-IV of Table 2, when changing the parameters in the DGP one at the time. The results in these tables should be compared to those in Table2.

The top panel of Table C1 shows results when there is no serial correlation in input prices. To do this we assume that the autoregressive parameters in the law of motion for the input prices are zero ($\lambda_{L_1} = \lambda_{M_1} = 0$). The bias in the OP estimator of α_M and α_K seems to be slightly smaller than the bias when input prices are serially correlated ($\lambda_{L_1} = \lambda_{M_1} = 0.8$ in Table 2), while the bias in the estimation of the labor elasticity is reduced significantly and it almost disappears. This suggests that the stronger the persistence in input prices the larger the bias in the estimated coefficients, at least for the labor coefficient. This makes intuitive sense because the correlation between input demands and $p_{jb} - \alpha_M p_{Mjb}$ is larger the stronger the serial correlation in input prices.

In the second panel of Table C1 we increase the variance of the shock to productivity by 50 percent, while in the third panel we decrease it by 50 percent (relative to the values in Table 2). This implies that input and output prices are more and less dispersed across firms, respectively. The magnitude of the OVB should therefore increase and decrease accordingly. This is exactly what occurs to the OP estimator in the second and third panels of Table C1.

Table C1: Other parameter configurations

	Low elasticity		High elasticity	
	I	II	III	IV
	No persistence of input prices ($\lambda_{L_1} = \lambda_{M_1} = 0$)			
α_M	0.6520	0.4001	0.4635	0.4000
	(0.0522)	(0.0014)	(0.0264)	(0.0011)
α_L	0.3999	0.4000	0.3774	0.4000
	(0.0286)	(0.0014)	(0.0226)	(0.0011)
α_K	-0.0891	0.2084	0.0191	0.2085
	(0.1648)	(0.2435)	(0.1311)	(0.2431)
	Larger variance of productivity shock ($1.5 \times \sigma_\epsilon^\omega$)			
α_M	0.7958	0.4000	0.5511	0.4000
	(0.0636)	(0.0015)	(0.0337)	(0.0013)
α_L	0.2146	0.4000	0.2670	0.4000
	(0.0593)	(0.0015)	(0.0319)	(0.0011)
α_K	-0.4128	0.2055	-0.1575	0.2055
	(0.2605)	(0.2278)	(0.2058)	(0.2276)
	Smaller variance of productivity shock ($0.5 \times \sigma_\epsilon^\omega$)			
α_M	0.5749	0.4000	0.4424	0.4000
	(0.0250)	(0.0016)	(0.0131)	(0.0013)
α_L	0.2526	0.4000	0.3462	0.4000
	(0.0236)	(0.0015)	(0.0125)	(0.0013)
α_K	0.1186	0.1916	0.1568	0.1960
	(0.0774)	(0.3656)	(0.064)	(0.3650)
Normalization	Yes	No	Yes	No
	OP	BL	OP	BL

See notes to Table 2. T = 15.